

# RF Filter Design Using Coupled Co-axial Resonators

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## Abstract

This paper describes the operation, design, implementation and performance of RF coupled resonator filters. It starts by considering discrete parallel inductor/capacitor resonators and then moves on to discuss the implementation of this style of filter at higher frequencies using short circuit co-axial resonators in place of the inductors.

## Introduction

Shunt mounted resonators, comprising a discrete inductor and capacitor in parallel; can be used to realise RF bandpass filters. Two or more resonators are normally coupled together to achieve the desired frequency response. This type of filter has found common use at lower RF frequencies in narrow band receivers for applications such as paging and PMR. As operating frequencies increase practical considerations result in it becoming increasingly difficult to realise such filters. One way of implementing this style of filter at higher frequencies is to use short circuit co-axial resonators in place of the inductors.

This paper starts by considering the fundamental performance of a single parallel inductor/capacitor (LC) resonator. It then describes how such resonators can be used to design RF bandpass filters. The implementation and achievable performance are then discussed and a means of extending the practical operating frequency range by the use of short-circuit co-axial resonators is detailed. Simulated performance and practical examples of coupled co-axial resonator filters are also presented.

## Single parallel LC resonator

A simple resonant circuit can be formed from a parallel inductor (L) and capacitor (C). The resonant frequency of such a structure is given by Equation 1. Below resonance this structure is predominantly inductive, above resonance it is predominantly capacitive and at resonance it is high impedance.

Equation 1 :

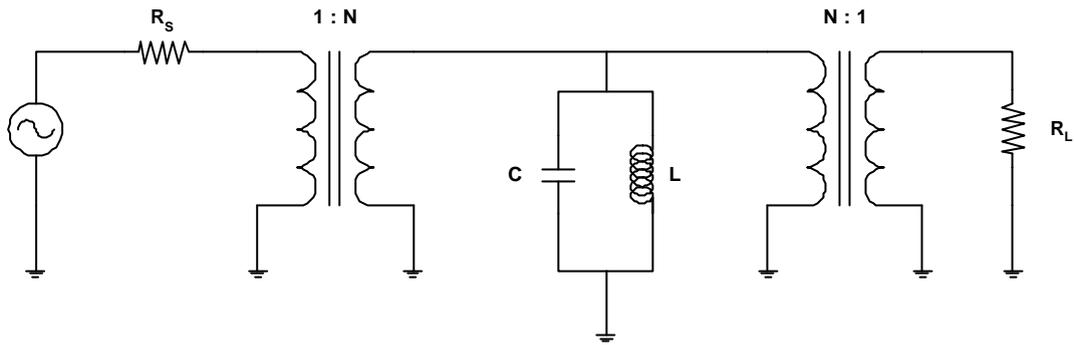
$$F_o = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$

The loaded Q of such a resonant structure is dependent on the load and source impedance and the Q of the inductor and capacitor themselves. If a single shunt-mounted resonator is considered, between a source of impedance  $R_s$  and a load of impedance  $R_L$ , then the loaded Q is given by Equation 2.

Equation 2 :

$$Q_L = \frac{R_p}{X_p}$$

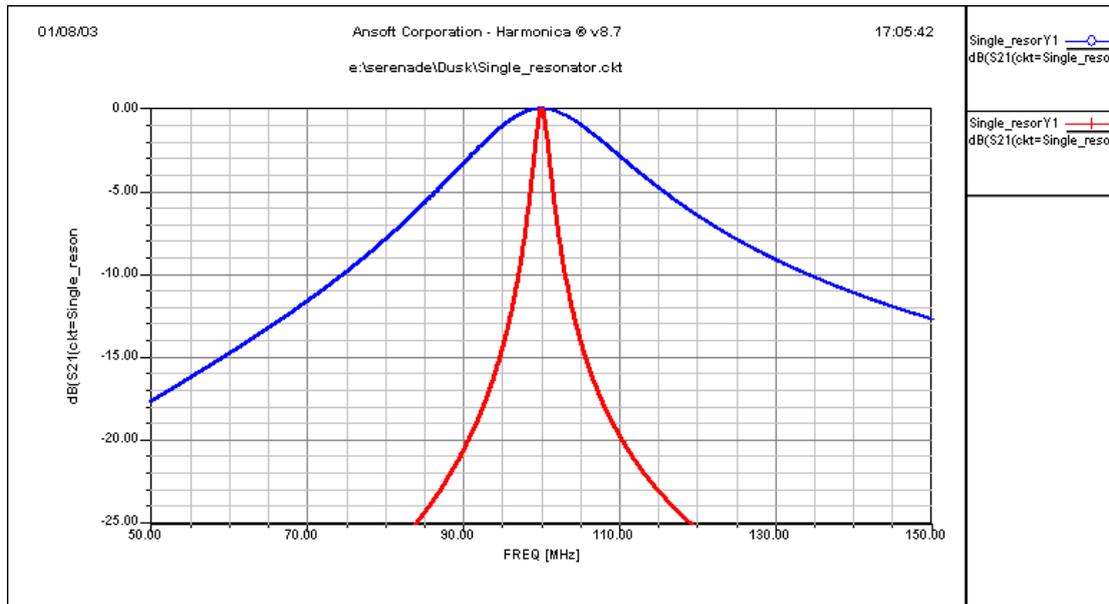
$R_p$  is the total equivalent parallel resistance of the source resistance, load resistance and the equivalent parallel resistance of the inductor and capacitor.  $X_p$  is the reactance of the inductor or capacitor at resonance. For source and load impedances of  $50\Omega$ , with practically realisable values of inductor and capacitor, the loaded Q of such a resonator would be low. Because the Q is also the ratio of the resonant frequency to the 3dB bandwidth, this means that the frequency selectivity would be low. The solution to this problem is to implement an impedance transformer that can shift the source and load impedance up in value such that the effective source and load impedance seen by the resonator is higher. This configuration is depicted in Figure 1.



**Figure 1: Single LC resonator with ideal impedance transformers**

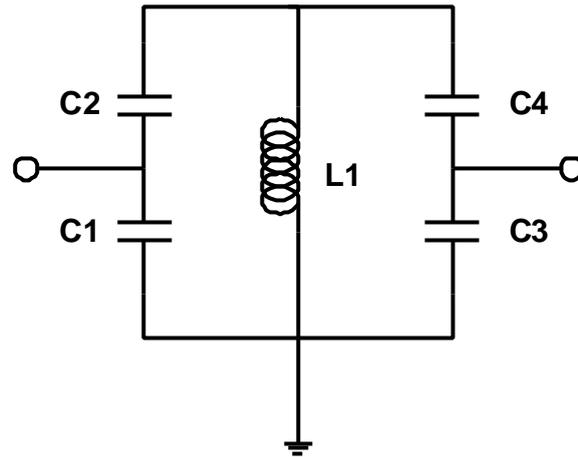
If an LC resonator were considered, with an inductor of 7.96nH and a capacitor of 318.3pF, the resonant frequency would be 100MHz. If this is shunt mounted between a 50Ω source and a 50Ω load the equivalent parallel resistance, assuming lossless reactive components, would be 25Ω and the loaded Q (from Equation 2) would be just 5. The insertion loss versus frequency for this case is the blue trace in Figure 2. The low Q results in a circuit that is not particularly frequency selective, having a 3dB bandwidth of 20MHz (the centre frequency divided by the loaded Q).

If impedance transformers are now included to transform the 50Ω source and load impedance up in value to 500Ω, the loaded Q of the circuit is increased to 50. The frequency response in this case is shown by the red trace in Figure 2 and its improved frequency selectivity is evident.



**Figure 2: Insertion loss of a single shunt mounted LC resonator with loaded Q of 5 compared to 50**

The wire-wound impedance transformers shown in Figure 1 are not normally used in practise. A more convenient way of realising the impedance transformation is to use a tapped capacitor or tapped inductor, the tapped capacitor usually being preferred for practical reasons. Figure 3 shows a single LC resonator with tapped capacitor transformers to transform the source and load impedances up in value. The total capacitance in parallel with L1 sets the resonant frequency, as defined in Equation 1.



**Figure 3: Single resonator with tapped capacitor transformers**

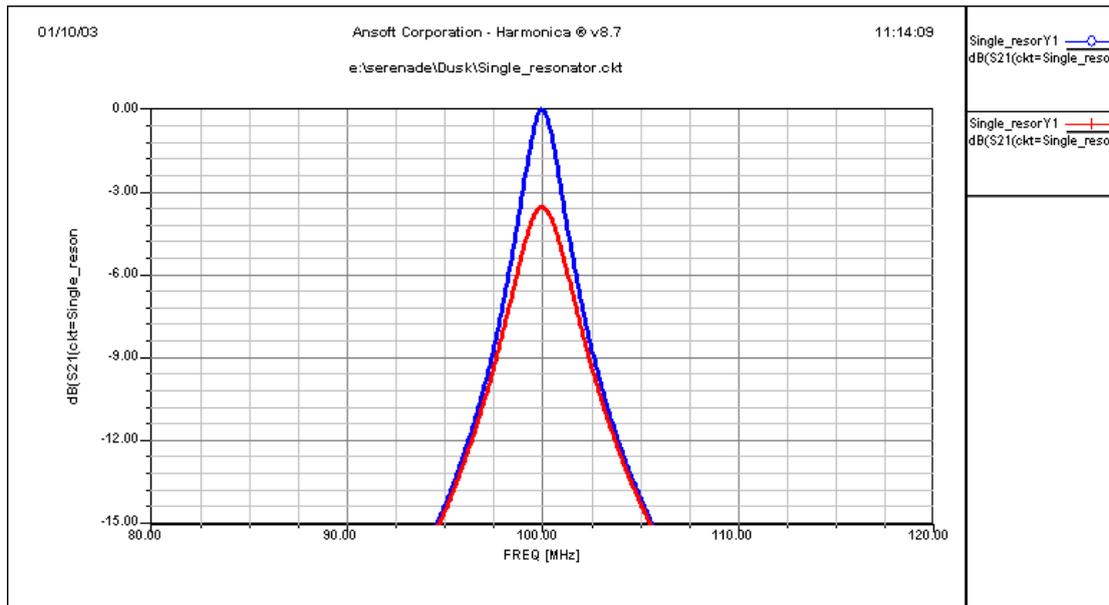
C1 and C2 perform an impedance transformation of the source and C3 and C4 perform an impedance transformation of the load. Equation 3 defines how the source impedance,  $R_S$ , is transformed to a higher value shunt resistance in parallel with L1. Some descriptions of this tapped capacitive transformer choose to neglect the second, frequency dependent term in Equation 3, however this approximation is not always valid.

Equation 3 :

$$R_{S_{eff}} = R_S \cdot \left( 1 + \frac{C_1}{C_2} \right)^2 + \frac{1}{(\omega \cdot C_2)^2 \cdot R_S}$$

The values of capacitor required to transform source and load impedances from  $50\Omega$  to  $500\Omega$  at a frequency of 100MHz, are  $C1=C3=502.9\text{pF}$  and  $C2=C4=232.84\text{pF}$ . The insertion loss versus frequency, assuming an ideal lossless 7.96nH inductor and lossless capacitors, is plotted as the blue trace in Figure 4. In practise the capacitors and inductors forming the resonator are not ideal but have losses and therefore a finite Q. This degrades the loaded Q of the resonator and increases the insertion loss at resonance. In most practical cases it is usually the Q of the inductor that dominates. Surface mount air core inductors are available with a Q of above 50 at 100MHz rising to above 100 at 1GHz.

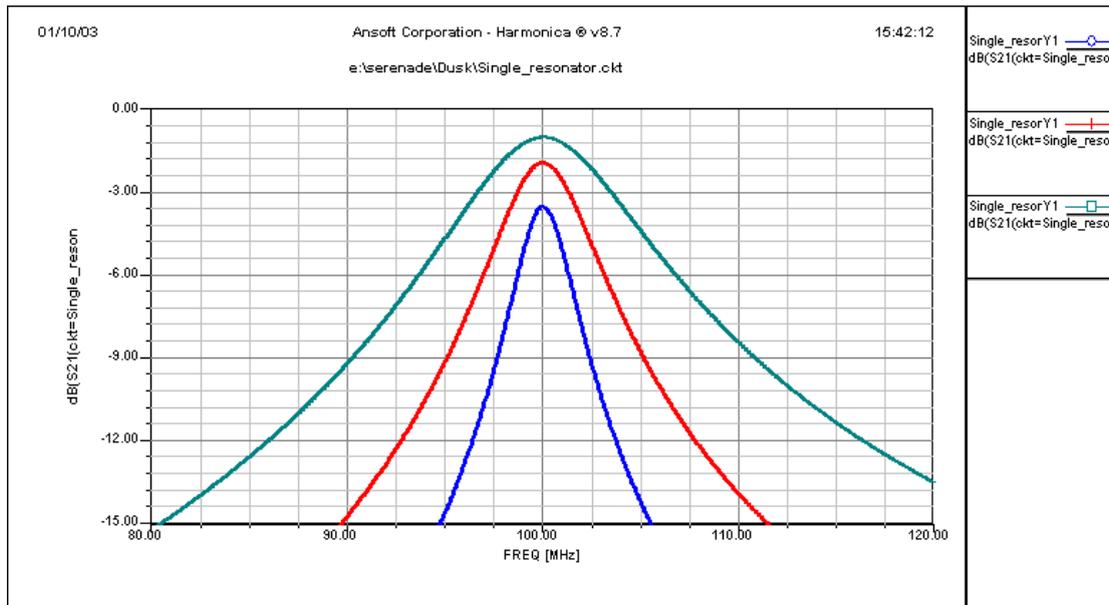
Considering the 100MHz resonator above, if the 7.96nH inductor had a Q of 100, the insertion loss versus frequency is plotted as the red trace in Figure 4. The loaded Q of the resonator has degraded to 33.3 and so the selectivity is reduced and the 3dB bandwidth is wider. In addition to this the insertion loss has increased to over 3dB. The reason for this increase in insertion loss is that a 7.96nH inductor with a Q of 100 at 100MHz, has an equivalent parallel resistance of  $500\Omega$  (Equation 2). As this is the same as the transformed source and load resistance it causes a significant increase in insertion loss. If the insertion loss of the resonator is to be kept low the equivalent parallel resistance of the inductor needs to be high in comparison to the source and load impedance.



**Figure 4: Frequency response of single resonator with tapped capacitor transformers. Ideal inductor (blue trace) and practical inductor (red trace).**

Increasing the value of the inductor can reduce the insertion loss of the resonator so long as its  $Q$  does not degrade. It can be seen from Equation 2 that for two inductors of the same unloaded  $Q$ , the higher value inductor will have a higher equivalent parallel resistance. The higher the equivalent parallel resistance, the lower the insertion loss at resonance. However, although the insertion loss may be lower the loaded  $Q$  of the entire resonant circuit (including the effects of the source and load impedances) will have reduced. This is because the larger value inductor means  $X_p$  will be higher (Equation 2) with  $R_p$  being dominated by the source and load impedances. Loss does not occur in the portion of  $R_p$  resulting from the source and load resistances, loss only occurs in the portion of  $R_p$  resulting from the finite  $Q$  of the inductor. In this way the insertion loss of the network can reduce whilst the loaded  $Q$  decreases.

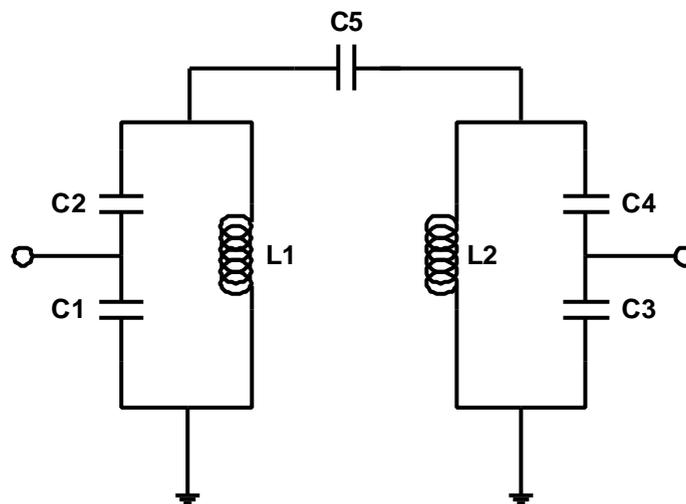
Figure 5 is a plot of the insertion loss versus frequency for a single LC resonator in a  $500\Omega$  system. The three traces reflect inductor values of  $7.96\text{nH}$ ,  $15.92\text{nH}$  and  $31.84\text{nH}$ . In all cases the inductor  $Q$  is 100 and the capacitor values have been modified to ensure resonance at  $100\text{MHz}$ . The trade-off can clearly be seen. As the inductor size increases the insertion loss gets lower but the loaded  $Q$  of the resonant circuit also reduces and so, therefore, does the frequency selectivity. At  $31.84\text{nH}$ , with a  $Q$  of 100, the inductor would have an equivalent parallel resistance of  $2\text{k}\Omega$ , which has much less effect on the insertion loss when shunt mounted in a  $500\Omega$  system. With any practical implementation it will be necessary to consider the appropriate trade-offs when selecting the appropriate inductor value.



**Figure 5: Frequency response of single resonator in 500Ω impedance. Inductor values of 7.96nH, 15.92nH and 31.84nH, each with a Q of 100.**

### Multiple Coupled LC Resonators

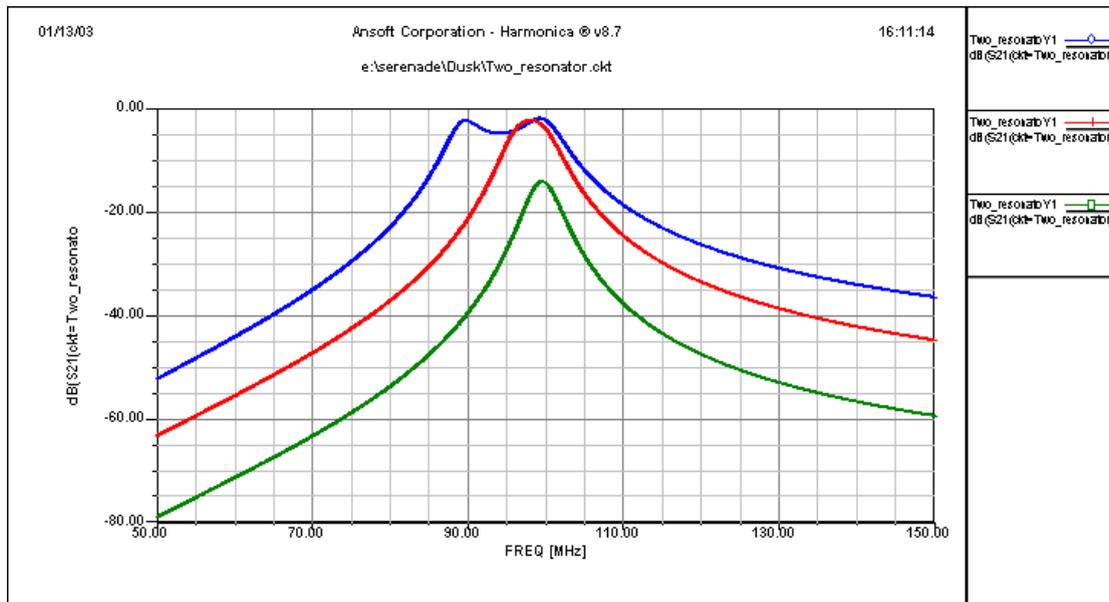
When realising a practical band pass filter, two or more resonators are often used. The resonators must be coupled together and the most common means of doing this is to use capacitors, as shown in Figure 6. The choice of coupling capacitor value is important. Too high a value results in over-coupling and a broadening of the response; too low a value results in under-coupling and excessive insertion loss.



**Figure 6: Schematic of a coupled resonator band pass filter**

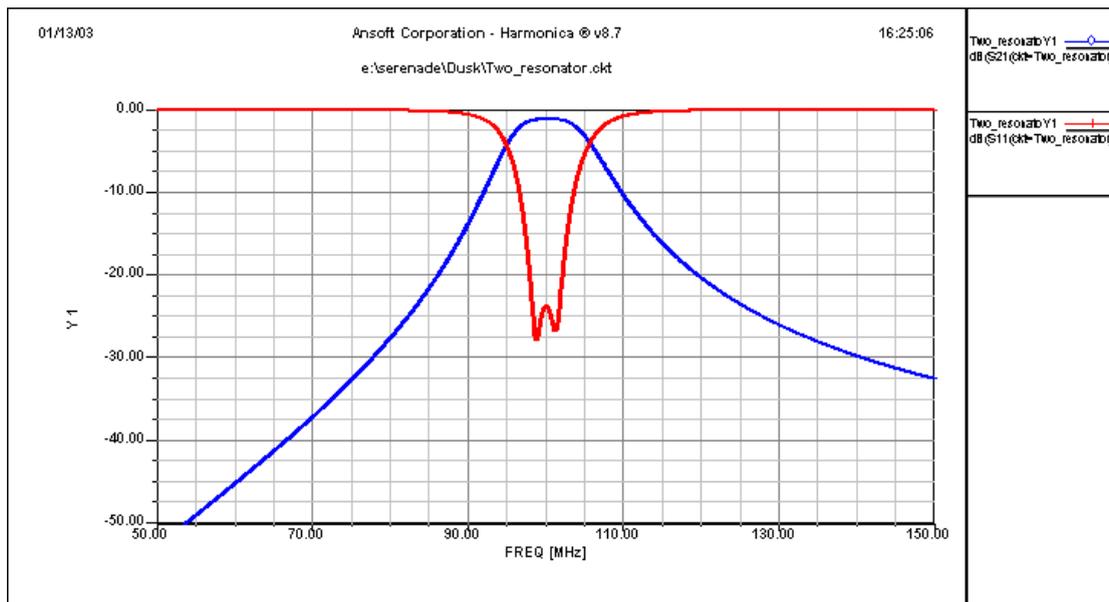
The effect of varying the value of the coupling capacitor is shown in Figure 7. This simulation assumes two resonators each using an inductor of 31.84nH, with a Q of 100 and tapped capacitor transformers to shift the load and source impedances up to 500Ω. The red trace is the critically coupled case, the green trace under coupled and the blue trace over coupled. In certain instances it can be useful to adopt an over-coupled design to tailor the filter bandwidth to that required. In practise it is normal to optimise the component values in the final stages of the design to achieve the desired performance.

In any practical implementation of this style of filter it is important that the grounding point for the shunt inductor and shunt capacitor is in close proximity.



**Figure 7: The effects of varying the degree of coupling**

The component values of the two-resonator design shown in Figure 6 were optimised to realise a filter with a 5% pass-band at 100MHz. The simulated performance of the optimised filter is shown in Figure 8. It is vital that component parasitics, such as the associated series inductance of chip capacitors and the effective parallel capacitance of inductors, are accounted for in the design process. This obviously becomes more important with increasing frequency as the parasitics have a more significant effect on the performance of the filter. It can be seen from the plot of simulated performance that the low side response of the filter rolls off more sharply than the high side response. This is a result of the high pass nature of the capacitive coupling elements.



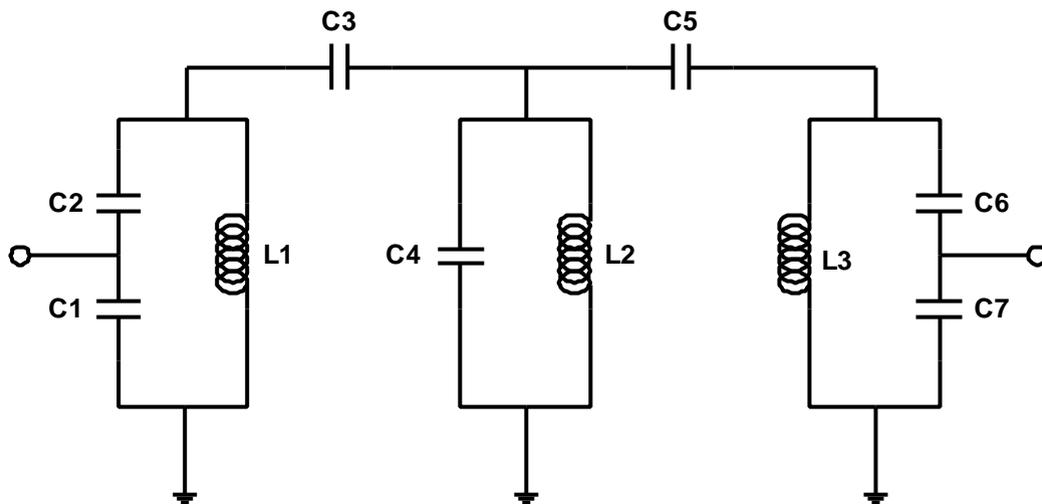
**Figure 8: Optimised performance of coupled LC resonator filter with two resonators**

It is also vital that the performance spread caused by the tolerance of the components to be used is assessed in the design phase. If the spread is deemed as unacceptable two approaches are open:

- Source more tightly toleranced components
- Include a degree of tunability

If a tunable filter is to be realised, this is most conveniently done by including some form of tunable capacitor in parallel with each resonator. The mid-range value of the tunable capacitor should be incorporated into the nominal filter design.

If a filter with greater out of band rejection is required, this can be achieved by cascading more resonators. Figure 9 shows the schematic of a three-resonator design.



**Figure 9: Coupled LC resonator filter schematic, three resonator design**

### **Replacing Inductors with Co-axial Resonators**

As operating frequencies increase it becomes increasingly difficult to realise coupled resonator filters using discrete inductors. In order to a resonant circuit with an adequate loaded Q (Equation 2), the required inductor value becomes very small. One answer to this problem is to use high Q, short-circuit co-axial resonators in place of the inductors. Figure 10 shows a photograph of a short-circuit co-axial resonator next to a coin for size comparison. They are available in typical lengths of between 4mm and 30mm. When the electrical length of the resonators is less than a quarter of a wavelength at the operating frequency, they appear inductive.



**Figure 10: Photograph of a co-axial resonator**

Co-axial resonators are available to a tight tolerance (0.5% length accuracy) with a relatively high unloaded  $Q$  (200 - 400). They are available with high dielectric constants ( $\epsilon_r$  values of between 10 and 90 are readily available), which means that even at modest RF frequencies the resultant filters can be quite compact. Examples of bandpass filter designs using short-circuit co-axial resonator are described below.

### **Coupled Co-axial Resonator Filter Design Example**

A bandpass filter was required with a pass-band covering 1710 to 1910MHz. This would allow both the GSM1800 and GSM1900 receive (downlink) bands to be passed. The application for this filter was in transceiver module that formed part of a multi-band GSM test set [1].

The circuit topology adopted for the filter is identical to the 3-element coupled LC resonator shown in Figure 9, except that the inductors were replaced with short-circuit co-axial resonators. Figure 11 shows the simulated response of the filter. The pass-band insertion loss is less than 0.7dB with a rejection of 60dB by 960MHz, which is the top of the GSM900 frequency allocation that must be rejected. The simulation includes practical values for the  $Q$  of the resonators and capacitors as well as all component and layout parasitics. A photograph of the practical implementation of this filter on the GSM test set transceiver PCB is shown in Figure 12. A photograph of the complete transceiver PCB, which includes a number of other coupled co-axial resonator filter designs, is shown in Figure 13.

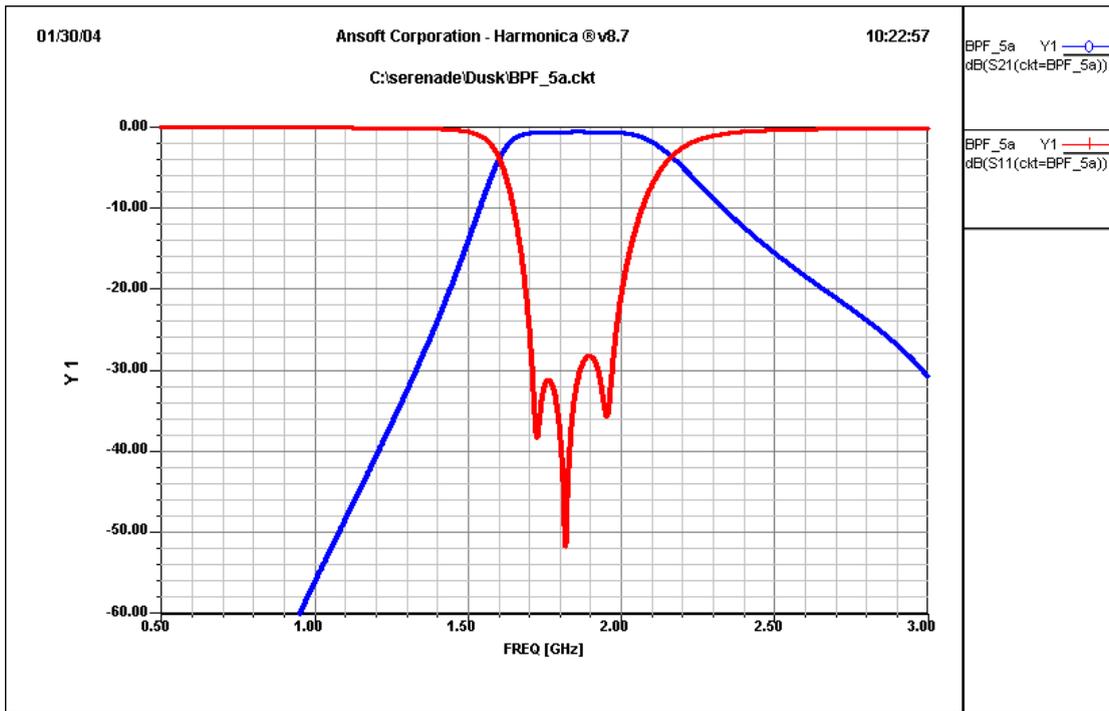


Figure 11: Simulated performance of Rx filter1

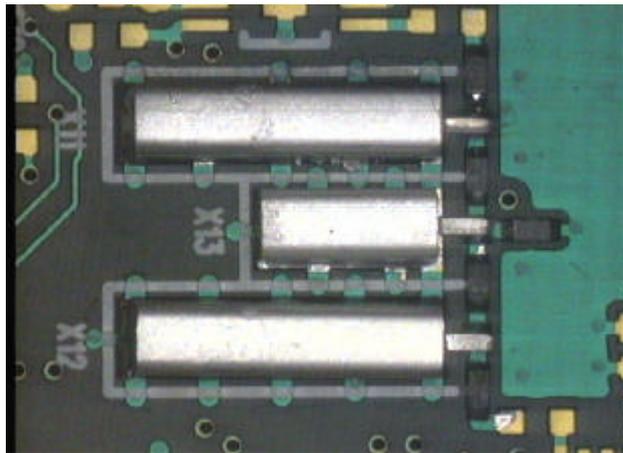


Figure 12: Photograph of Rx filter1



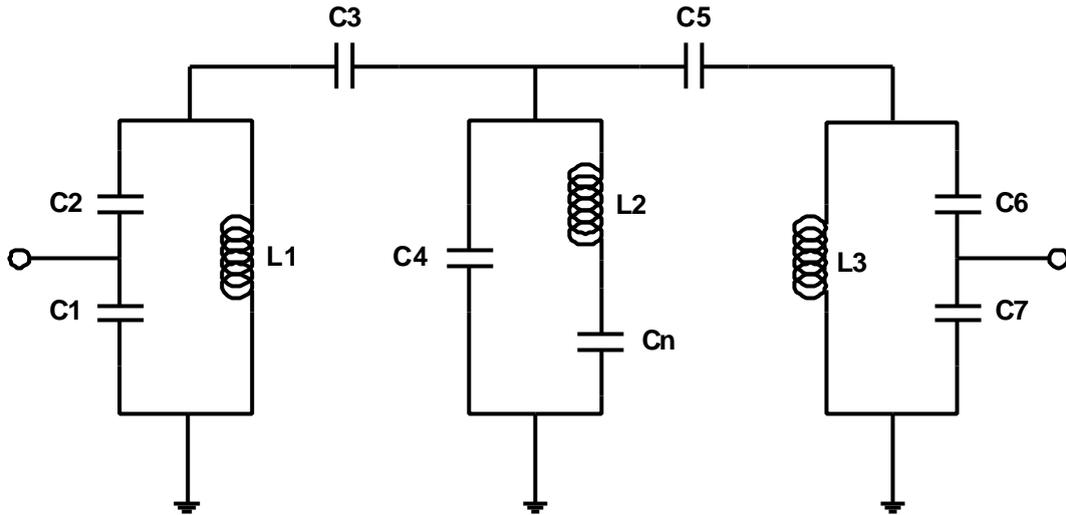
**Figure 13: Photograph of the complete transceiver module**

### **Incorporating a Rejection Notch into the Filter Response**

Bandpass filters are designed to have low insertion loss in the pass-band and to provide adequate rejection in the stop-band. For a given filter topology, the way to provide more rejection is to increase the order of the filter. With the coupled resonator filters described here, this means adding more resonators. However, adding more resonators increases pass-band insertion loss, size, parts count and cost.

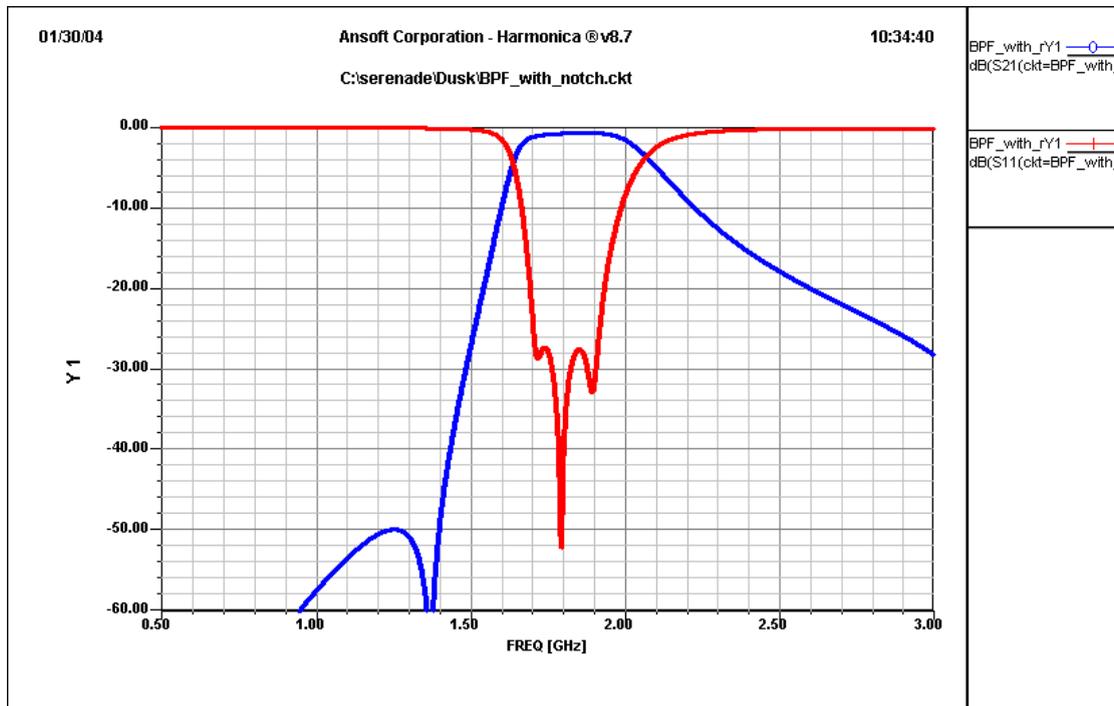
In most systems, there are certain known frequencies that must be rejected to an adequate degree to avoid performance degradation and/or specification failure. In communications systems these could be the image frequency, the LO frequency or the operating frequency of a problematic close-to-band signal generated by another co-located system. It is not unusual for the rejection required at one of these frequencies to dominate the overall stop-band rejection requirement. One simple method of avoiding designing a high order filter to satisfy these rejection requirements is to include a “notch” in the frequency response of the filter by incorporating a transmission zero at the required rejection frequency.

If the three resonator filter of Figure 9 is considered, a below band notch can be added by including a capacitor in series with one of the shunt inductors, as depicted in the schematic of Figure 14. In this case  $C_n$  will resonate with  $L_2$  (at the frequency given in Equation 1) to realise a shunt short-circuit. In theory this is a transmission zero at that frequency and in practise it will result in a “notch” of increased attenuation. The  $Q$  of  $L_2$  and  $C_n$  determines the width and depth of the notch. If the notch frequency is far enough below the pass-band then the value of  $C_n$  will be high enough to provide a low reactance path in the pass-band and the insertion loss will be largely unaffected. As before, coupled co-axial resonator versions of this filter, suitable for higher frequency operation, can be realised by replacing the inductors of Figure 14 with short-circuit coaxial resonators.



**Figure 14: Schematic of a coupled LC resonator filter, with below band notch**

If we now consider the 1710 to 1910MHz coupled coaxial resonator bandpass filter design described above but also wish to incorporate a rejection notch at 1350MHz, this can be achieved by including a capacitor ( $C_n$ ) in series with the central shunt resonator. Although the selection of the correct capacitor value for  $C_n$  is all that is required to produce a notch at the required frequency, in practise it is also necessary to optimise the values of the other components to also maintain an adequate pass-band response. The simulated performance of the optimised filter response, with stop-band notch at 1350MHz, is shown in Figure 15.



**Figure 15: Simulated performance of bandpass filter with notch at 1350MHz**

The inclusion of the notch has caused a very slight increase in insertion loss, from 0.7dB to 0.8dB. However, the rejection at 1350MHz has increased from around 30dB to around 60dB. This clearly demonstrates the value of this technique for achieving significant increases in rejection at problematic frequencies for a very modest increase in circuit complexity and very little sacrifice in pass-band performance. It is also worth noting the slight degradation in the

stop-band rejection above the pass-band. Attenuation at 3GHz has decreased from 31dB to 28dB; this is a result of the fact that although the reactance of  $C_n$  is low in and above the pass-band, it is still finite.

### **Summary and Conclusions**

The operation of coupled resonator filters has been described and the issues affecting their practical implementation have been discussed. The use of short-circuit co-axial resonators to increase the practical operating frequency of such filters has been presented and design examples have been given. The techniques described can be used to realise relatively narrow band RF filters at frequencies up to several GHz. The practical examples given in this paper were used in the transceiver module of the multi-band GSM test set described in [1].

### **References**

- [1] Steve Williamson, "The RF Development of a Multi-band GSM Test Set", Microwave Engineering Europe, March 2004